Sesión Especial 9

Kinetic equations and applications

Organizadores
- José A. Cañizo (Universidad de Granada)
- Sara Merino-Aceituno (Universidad de Viena)
- Iván Moyano (University of Cambridge)

Descripción

Las ecuaciones cinéticas juegan un papel crucial en la derivación de ecuaciones macroscópicas (típicamente ecuaciones en derivadas parciales) a partir de ecuaciones microscópicas (correspondientes, típicamente, a procesos discretos que se pueden describir a través de procesos de Markov o ecuaciones diferenciales estocásticas). Este tipo de ecuaciones establecen un puente entre descripciones microscópicas y macroscópicas de un sistema dado, proporcionando la evolución a lo largo del tiempo de la distribución de las partículas o agentes discretos de la descripción microscópica.

Esta sesión especial tiene como objetivo representar la investigación puntera actual y sus interacciones con varias sub-disciplinas: análisis matemático de las ecuaciones, cálculo numérico, modelización y aplicaciones.

Programa

JUEVES, 7 de febrero (tarde)

15:30 – 16:00	Matteo Bonforte (Universidad Autónoma de Madrid) *Extinction Rates for Fast Diffusion Equations on Generic Bounded Domains*
16:00 – 16:30	María José Cáceres (Universidad Granada) *From blow-up to global existence for the Nonlinear Noisy Leaky Integrate and Fire models describing neural networks*
16:30 – 17:00	Matías Delgadino (Imperial College London) *Propagation of Chaos by Gamma-convergence*
17:30 – 18:00	Susana Gutiérrez (Universidad de Birmingham, Reino Unido) *Strichartz estimates for the kinetic transport equation*
18:00 – 18:30	Fabricio Maciá (Universidad Politécnica de Madrid) *Propagation of a quantum particle in the disk*

Viernes, 8 de febrero (mañana)

9:30 – 10:00	Fernando Quirós (Universidad Autónoma de Madrid) *Anisotropic nonlocal diffusion equations with singular forcing*
10:00 – 10:30	Havva Yoldaş (BCAM / Universidad de Granada) *Hypocoercivity of linear kinetic equations via Harris' theorem*
10:30 – 11:00	Iván Moyano (Universidad de Cambridge) *Carleman inequalities for the linear Boltzmann equation and some applications*
11:30 – 12:00	José Alfredo Cañizo (Universidad de Granada) *Energy estimates for nonlocal diffusion equations*
12:00 – 12:30	Sara Merino Aceituno (Universidad de Viena) *A new flocking model through body attitude coordination*

Extinction Rates for Fast Diffusion Equations on Generic Bounded Domains

Matteo Bonforte

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Abstract. We investigate the homogeneous Dirichlet problem for the Fast Diffusion Equation $u_t = \Delta u^m$, posed in a smooth bounded domain $\Omega \subset \mathbb{R}^N$, in the exponent range $m_s = (N-2)_+/(N+2) < m < 1$. It is known that bounded positive solutions $u(t,x)$ of such problem extinguish in a finite time $T = T(u_0)$ and also that they approach a separate variable solution $u(t,x) \sim (T-t)^{1/(1-m)} S(x)$, as $t \to T^-$. It has been shown in 2012 that $v(x,t) = u(t,x)(T-t)^{-1/(1-m)}$ tends to $S(x)$ as $t \to T^-$, uniformly in the relative error norm; to our knowledge, this is the best asymptotic result valid in the whole range of parameters. Starting from this result, we investigate the fine asymptotic behaviour, which amounts to derive (sharp) rates of convergence for the relative error. The proof is based on a new entropy method and one of the fundamental ingredients of such method is a (improved) weighted Poincaré inequality, that we show to be true on a generic bounded domain. Another essential aspect of the method is the new concept of "almost orthogonality", which can be thought as a nonlinear analogous of the classical orthogonality condition needed to obtain improved Poincaré inequalities and sharp convergence rates for linear flows.

Joint work with Alessio Figalli

From blow-up to global existence for the Nonlinear Noisy Leaky Integrate and Fire models describing neural networks

María José Cáceres

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Abstract. In this talk we analyse nonlinear noisy leaky integrate and fire (NNLIF) models, which describe the activity of neural networks by means of the membrane potential. These models are based on nonlinear systems of PDEs of Fokker-Planck type. We study the wide range of phenomena that appear in this kind of models: blow-up/global existence, asynchronous/synchronous solutions, instability/stability of the steady states.

Referencias

[1] M. J. Cáceres, J. A. Carrillo, and B. Perthame, Analysis of nonlinear noisy integrate & fire neuron models: blow-up and steady states, Journal of Mathematical Neuroscience, 1-7 (2011).

[2] M. J. Cáceres and B. Perthame, Beyond blow-up in excitatory integrate and fire neuronal networks: refractory period and spontaneous activity, Journal of theoretical Biology, 350 (2014), pp. 81–89.

[3] M. J. Cáceres, P. Roux, D. Salort, and R. Schneider, Global-in-time classical solutions and qualitative properties for the nnlif neuron model with synaptic delay, arXiv preprint arXiv:1806.01934, (2018).

[4] M. J. Cáceres and R. Schneider, Blow-up, steady states and long time behaviour of excitatory-inhibitory nonlinear neuron models, Kinetic and Related Models, 10 (2017), pp. 587–612.

[5] M. J. Cáceres and R. Schneider, Analysis and numerical solver for excitatory-inhibitory net- works with delay and refractory periods, ESAIM: Mathematical Modelling and Numerical Analysis, 52 (2018), pp. 1733–1761.

Joint work with Carrillo, Perthame, Roux, Schneider and Salort

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Propagation of Chaos by Gamma-convergence

Matías Delgadino

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Abstract. We show the classical result of propagation of chaos for the Mc-Kean–Vlasov equation using the framework of gradient flows of probability valued probability measures. The main contribution of this talk is to clearly identify the framework of convergence of the finite dimensional Wasserstein flow into the infinite dimensional one. The rigorous proof relies on the uniform convexity of the finite dimensional flow, which coincides with the classical result of propagation of chaos.

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Strichartz estimates for the kinetic transport equation

Susana Gutiérrez

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Abstract. The aim of this talk is to present some of the recent progress on the study of Strichartz estimates for the kinetic transport equation. Here, the term Strichartz estimate refers to a space-time estimate for either the solution of the homogeneous kinetic equation or a relevant quantity associated to the solution. In particular, we consider the velocity averaging operator of the solution and discuss new sharp smoothing estimates in the framework of hyperbolic Sobolev spaces, when the velocity domain is either the sphere or the unit ball.

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Propagation of a quantum particle in the disk

Fabricio Maciá

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Abstract. The simplest model for the propagation of a quantum particle confined in a disk in Euclidean space is the Schrödinger evolution associated to the Dirichlet Laplacian. Eugene Wigner showed in the 30s' that this equation can be reformulated as a transport kinetic-type equation for a space-momentum density with a non-local term that accounts for the collisions with the boundary and the action of the electric potential. In this talk we will present recent work in collaboration with Nalini Anantharaman and Matthieu Léautaud in which we give a precise description to the solutions of this equation in the high-frequency regime. More precisely, we show that the dynamics of Wigner equation can be described in the small wavelength limit as a superposition of one-dimensional Schrödinger evolutions on the invariant tori of the classical billiard flow. I will also present applications to the quantification of dispersive and unique continuation type principles for the Schrödinger equation.

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Anisotropic nonlocal diffusion equations with singular forcing

Fernando Quirós

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Abstract. We prove existence, uniqueness and regularity of solutions of nonlocal heat equations associated to anisotropic stable diffusion operators. The main features are that the right-hand side has very few regularity and that the spectral measure can be singular in some directions. The proofs require having good enough estimates for the corresponding heat kernels and their derivatives. The results are expected to be useful to obtain higher regularity for nonlinear diffusion problems of this kind.

Joint work with Arturo de Pablo and Ana Rodríguez.

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Hypocoercivity of linear kinetic equations via Harris' theorem

HAVVA YOLDAS

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Abstract. In this talk, we study convergence to equilibrium of the linear relaxation Boltzmann (linear BGK) and the linear Boltzmann equations by using a method from the theory of Markov processes known as Harris's theorem. We consider equations either on the torus or on the whole space with a confining potential. We give explicit convergence results in total variation or weighted total variation norms and the convergence rates are exponential when we consider the equations on torus or on the whole space with a confining potential growing at least quadratically at infinity. Moreover, we give algebraic convergence rates when subquadratic potentials considered.

Referencias

[1] Cañizo , Qui Cao, Evans and Yoldas, Hypocoercivity of linear kinetic equations via Harris's Theorem, Journal, In preparation

Joint work with José A. Cañizo (U. Granada), Chu Qui Cao (Paris-Dauphine) and Josephine Evans (Paris-Dauphine).

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Carleman inequalities for the linear Boltzmann equation and some applications

IVÁN MOYANO

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Abstract. In this talk we investigate the problem of deriving Carleman inequalities, a weighted energy estimate useful in the study of unique continuation properties, for the full linear Boltzmann operator. This problem is a first step to understand how to extend the classical theory of Carleman inequalities (due to Hormander, among others), initially conceived to deal with differential operators, to some partially nonlocal operators. As a result, we can use the resulting Carleman inequalities to study an inverse problem (determination of coefficients from boundary measurements) for the linear Boltzmann equation.

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Energy estimates for nonlocal diffusion equations

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Abstract. We present an inequality involving entropies and their production for nonlocal equations of the form $\partial_t u = J * u - u$, where J is a smooth kernel with fast decay. Using it we show decay rates of this nonlocal equation in H^k norms, and we are able to extend these results to other linear, nonlocal diffusions with a more complex behaviour.

Joint work with A. Molino.

A new flocking model through body attitude coordination

Sara Merino-Aceituno

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Abstract. We present a new model for multi-agent dynamics where each agent is described by its position and body attitude: agents travel at a constant speed in a given direction and their body can rotate around it adopting different configurations. Agents try to coordinate their body attitudes with the ones of their neighbours. This model is inspired on the Vicsek model. The goal of this talk will be to present this new flocking model, its relevance and the derivation of the macroscopic equations from the particle dynamics.